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BOSE SYMMETRY AND CHIRAL DECOMPOSITION OF 2D FERMIONIC DETERMINANTS [1]

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Abstract

We show in a precise way, either in the fermionic or its bosonized version, that Bose symmetry provides a systematic way to carry out the chiral decomposition of the two dimensional fermionic determinant. Interpreted properly, we show that there is no obstruction of this decomposition to gauge invariance, as is usually claimed. Finally, a new way of interpreting the Polyakov-Wiegman identity is proposed.

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It is often claimed[1] that the chiral decomposition of the two dimensional fermionic determinant poses an obstruction to gauge invariance. In this paper we clarify several aspects of this decomposition. Contrary to the usual approach, the inverse route, whereby two chiral components are fused or soldered, is also examined in details. A close correspondence between the splitting and the soldering processes is established. By following Bose symmetry it is possible to give explicit expressions for the chiral determinants which show, in both these procedures, that there is no incompatibility with gauge invariance at the quantum level. Two important consequences emerging from this analysis are the close connection between Bose symmetry and gauge invariance, and a novel interpretation of the Polyakov-Wiegman identity[2].

It is worth mentioning that understanding the properties of 2D-fermionic determinats and the associated role of Bose symmetry is crucial because of several aspects. For instance, the precise form of the one cocycle necessary in the recent discussions on smooth functional bosonisation [3, 4] is only dictated by this symmetry [5]. Furthermore this cocycle, which is just the 2D anomaly, is known to be the origin of anomalies in higher dimensions by a set of descent equations [6]. Incidentally, the anomaly phenomenon still defies a complete explanation.

To briefly recapitulate the problem of chiral decomposition, consider the vacuum functional,

$$\begin{aligned} e^{iW[A]} &= \int d\bar{\psi}\, d\psi \exp\{i \int d^2x\, \bar{\psi}(i\partial\!\!\!/ + eA\!\!\!/)\psi\} \\ &= \det(i\partial\!\!\!/ + eA\!\!\!/) \end{aligned} \tag{1}$$

where the expression for the determinant follows immediately by imposing gauge invariance,

$$W[A] = N \int d^2x\, A_\mu \Pi^{\mu\nu} A_\nu \tag{2}$$

with $\Pi^{\mu\nu} = g^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\Box}$; $\mu, \nu = 0, 1$ being the transverse projector. An explicit one loop calculation yields [7] $N = \frac{e^2}{2\pi}$. Introducing light-cone variables,

$$A_\pm = \frac{1}{\sqrt{2}}(A_0 \pm A_1) = A^\mp \quad ; \quad \partial_\pm = \frac{1}{\sqrt{2}}(\partial_0 \pm \partial_1) = \partial^\mp \tag{3}$$

with the projector matrix given by,

$$\Pi^{\mu\nu} = \frac{1}{2}\begin{pmatrix} -\frac{\partial_-}{\partial_+} & 1 \\ 1 & -\frac{\partial_-}{\partial_+} \end{pmatrix} \tag{4}$$

it is simple to rewrite (2) as,

$$W[A_+, A_-] = -\frac{N}{2}\int d^2x \, \{A_+ \frac{\partial_-}{\partial_+} A_+ + A_- \frac{\partial_+}{\partial_-} A_- - 2A_+ A_-\} \tag{5}$$

The factorization of (1) into its chiral components yields,

$$\det(i\partial\!\!\!/ + eA\!\!\!/) = \det(i\partial\!\!\!/ + eA\!\!\!/_+)\det(i\partial\!\!\!/ + eA\!\!\!/_-) \tag{6}$$

where $A\!\!\!/_\pm = A\!\!\!/ P_\pm$ with the chiral projector defined as $P_\pm = \frac{1 \pm \gamma_5}{2}$. The effective action for the vector theory in terms of the chiral components is now obtained from (6), leading to an effective action,

$$\begin{aligned} W_{eff} &= -\frac{N}{2}\int d^2x \, \{A_+ \frac{\partial_-}{\partial_+} A_+ + A_- \frac{\partial_+}{\partial_-} A_-\} \\ &= W[A_+, 0] + W[0, A_-] \end{aligned} \tag{7}$$

which does not reproduce the expected gauge invariant result (5). The above factorization is therefore regarded as an obstruction to gauge invariance.

It is important to notice that (7) follows from (6) only if one naively computes the chiral determinants from the usual vector case (5) by substituting either $A_+ = 0$ or

$A_- = 0$. This may be expected naturally since $\det(i\not\partial + e\not A) = \det(i\not\partial + e\not A_+ + e\not A_-)$. But the point is that whereas the usual Dirac operator has a well defined eigenvalue problem,

$$\not D\psi_n = (i\not\partial + e\not A)\psi_n = \lambda_n\psi_n \tag{8}$$

with the determinant being defined by the product of its eigenvalues, this is not true for the chiral pieces in the RHS of (6), which lacks a definite eigenvalue equation [8, 5] because the kernels map from one chiral sector to the other,

$$\not D_\pm\psi_\pm = \lambda\psi_\mp \tag{9}$$

with $\psi_\pm = P_\pm\psi$. Consequently, it is not possible to interpret, however loosely or naively, any expression obtainable from $\det \not D$ by setting $A_\pm = 0$, as characterising $\det \not D_\mp$.

Since $\det \not D_\pm$ are not to be regarded as $W[A_+, 0]$ or $W[0, A_-]$ in (7), it is instructive to clarify the meaning of the latter expressions. Reconsidering $\det \not D$ as $\det(i\not\partial + e\not A_+ + e\not A_-)$ it is easy to observe that the fundamental fermion loop decomposes into four pieces **(see figure)**.

At the unregularized level there are different choices of interpreting these diagrams, depending on the location of the chiral projectors $P_\pm$. In particular, by pushing one of these projectors through the loop and inserting it at the other vertex would yield vanishing contributions for the last two diagrams, since $P_+P_- = 0$. It was shown earlier by one of us [5], in a different context, that Bose symmetry provided a definite guideline in manipulating such diagrams. In other words, the position of the projectors is to be preserved exactly as appearing above, and the contributions explicitly computed from (2) by appropriate replacements at the vertices. This procedure implies a consistent way of regularizing all four graphs. Thus,

$$
\begin{aligned}
W_{1(2)} &= N\int d^2x\; A_\mu \mathcal{P}^{\mu\nu}_{+(-)} \Pi_{\nu\alpha} \mathcal{P}^{\alpha\beta}_{-(+)} A_\beta \\
W_{3(4)} &= N\int d^2x\; A_\mu \mathcal{P}^{\mu\nu}_{+(-)} \Pi_{\nu\alpha} \mathcal{P}^{\alpha\beta}_{+(-)} A_\beta
\end{aligned} \tag{10}
$$

where

$$
\mathcal{P}^{\alpha\beta}_{+(-)} = \frac{1}{2}(g^{\alpha\beta} \pm \epsilon^{\alpha\beta}) \quad ; \epsilon^{+-} = \epsilon_{-+} = 1 \tag{11}
$$

Using (4) it is easy to simplify (10) as,

$$
W_1 = W[A_+, 0] \;\; , \;\; W_2 = W[0, A_-] \;\; , \;\; W_3 = W_4 = \frac{N}{2} A_+ A_- \tag{12}
$$

Adding all four terms exactly reproduces the gauge invariant result (5). If, on the contrary, Bose symmetry was spoilt in the last two graphs as indicated earlier so that $W_3 = W_4 = 0$, the gauge noninvariant structure (7) is obtained. This shows the close connection between Bose symmetry and gauge invariance. Recall that the same is also true in obtaining the ABJ anomaly from the triangle graph[9, 10]. Furthermore $W[A_+, 0]$ and $W[0, A_-]$ are now seen to correspond to graphs W_1 and W_2, respectively, evaluated in a very specific fashion. It is also evident that the incorrect manner of abstracting $\det(i\partial\!\!\!/ + eA\!\!\!/_\pm)$ from $\det(i\partial\!\!\!/ + eA\!\!\!/)$ violates Bose symmetry leading to an apparent contradiction between chiral factorization and gauge invariance. Consequently the possibility of ironing out this contradiction exists by interpreting the chiral determinants as,

$$
\begin{aligned}
-\,i \ln\det(i\partial\!\!\!/ + eA\!\!\!/_+) &= W[A_+, 0] + \frac{N}{2}\int d^2x\; A_+ A_- \\
-i \ln\det(i\partial\!\!\!/ + eA\!\!\!/_-) &= W[0, A_-] + \frac{N}{2}\int d^2x\; A_+ A_-
\end{aligned} \tag{13}
$$

These expressions just reduce to the naive definitions if the crossing graphs are ignored or, equivalently, Bose symmetry is violated.

To put (13) on a solid basis it must be recalled that (6), as it stands, is only a formal identity. A definite meaning can be attached provided some regularization is invoked to explicitly define the determinants appearing on either side of the equation. Using a regularization that preserves the vector gauge symmetry of the LHS of (6) led to the expression (5). As is well known [6, 11] there is no regularization that retains the chiral symmetry of the pieces in the RHS of (6). An explicit one loop computation yields [11, 12], in a bosonized language,

$$\begin{aligned} W_+[\varphi] &= \frac{1}{4\pi}\int d^2x\,\Big(\partial_+\varphi\partial_-\varphi + 2\,e\,A_+\partial_-\varphi + a\,e^2\,A_+A_-\Big) \\ W_-[\rho] &= \frac{1}{4\pi}\int d^2x\,\Big(\partial_+\rho\partial_-\rho + 2\,e\,A_-\partial_+\rho + b\,e^2\,A_+A_-\Big) \end{aligned} \tag{14}$$

where a and b are parameters manifesting regularization, or equivalently, bosonization ambiguities. It is simple to verify that a straightforward application of the usual bosonization rules: $\bar\psi i\partial\!\!\!/\psi \to \partial_+\varphi\partial_-\varphi$ and $\bar\psi\gamma_\mu\psi \to \frac{1}{\sqrt{\pi}}\epsilon_{\mu\nu}\partial^\nu\varphi$, which are valid *only* when the vector gauge symmetry is preserved, would just reproduce (14) with $a = b = 0$. Subsequently, by functionally integrating out the scalar fields φ and ρ, exactly yields the two pieces $W[A_+, 0]$ and $W[0, A_-]$ given in (7), which is what one obtains by simply putting $A_\pm = 0$ directly into the expressions for the vector determinant. This reconfirms the invalidity of identifying the chiral determinants by naively using rules valid for the vector case.

We now show precisely how two independent chiral components (14) are soldered to yield the LHS of (6). This idea of soldering was initially introduced by Stone [13] and recently exploited by one of us [14] in a different context. It consist in lifting the gauging of a global symmetry to its local version. Let us then consider the gauging

of the following global symmetry of (14)

$$\begin{aligned} \delta\varphi &= \delta\rho = \alpha \\ \delta A_{\pm} &= 0 \end{aligned} \tag{15}$$

Then it is found from (14) that

$$\begin{aligned} \delta W_{+}[\varphi] &= \int d^2x\, \partial_{-}\alpha\; J_{+}(\varphi) \\ \delta W_{-}[\rho] &= \int d^2x\, \partial_{+}\alpha\; J_{-}(\rho) \end{aligned} \tag{16}$$

where,

$$J_{\pm}(\eta) = \frac{1}{2\pi}(\partial_{\pm}\eta + \, e\, A_{\pm}) \quad ; \quad \eta = \varphi, \rho \tag{17}$$

Next, introduce the soldering field $B_{\pm}$ so that,

$$W_{\pm}^{(1)}[\eta] = W_{\pm}[\eta] - \int d^2x\, B_{\mp}\, J_{\pm}(\eta) \tag{18}$$

Then it is easy to verify that the modified action,

$$W[\varphi, \rho] = W_{+}^{(1)}[\varphi] + W_{-}^{(1)}[\rho] + \frac{1}{2\pi}\int d^2x\, B_{+}\, B_{-} \tag{19}$$

is invariant under an extended set of transformations that includes (15) together with,

$$\delta B_{\pm} = \partial_{\pm}\alpha \tag{20}$$

Using the equations of motion, the auxiliary soldering field can be eliminated in favour of the other variables,

$$B_{\pm} = 2\pi J_{\pm} \tag{21}$$

so that the soldered effective action derived from (19) reads,

$$W[\Phi] = \frac{1}{4\pi}\int d^2x \left\{ \Big(\partial_+\Phi\partial_-\Phi + 2\,e\,A_+\partial_-\Phi - 2\,e\,A_-\partial_+\Phi \Big) + (a+b-2)\,e^2\,A_+\,A_- \right\} \tag{22}$$

where,

$$\Phi = \varphi - \rho \tag{23}$$

We may now examine the variation of (22) under the lifted gauge transformations, $\delta\varphi = \delta\rho = \alpha$ and $\delta A_{\pm} = \partial_{\pm}\alpha$, induced by the soldering process. Note that this is just the usual gauge transformation. It is easy to see that the expression in parenthesis is gauge invariant, and by functionally integrating out the Φ field one verifies that it reproduces (5). Thus, the soldering process leads to a gauge invariant structure for W provided

$$a + b - 2 = 0 \tag{24}$$

It might appear that there is a whole one parameter class of solutions. However Bose symmetry imposes a crucial restriction. Recall that in the Feynman graph language this symmetry was an essential ingredient in preserving compatibility between gauge invariance and chiral decomposition. In the soldering process, this symmetry, which is just the left-right (or $+-$) symmetry in (14), is preserved with $a = b$. Coupled with (24) this fixes the parameters to unity and proves our assertion announced in (13). It may be observed that the soldering process can be carried through for the nonabelian theory as well, and a relation analogous to (22) is obtained[3].

[3] see appendix

An alternative way of understanding the fixing of parameters is to recall that if a Maxwell term is included in (13) to impart dynamics, then this corresponds to the chiral Schwinger model[11]. It was shown that unitarity is violated unless a(or b)≥ 1. Imposing (24) immediately yields $a = b = 1$ as the only valid answer, showing that the bound gets saturated. It is therefore interesting to note that (24) together with unitarity leads naturally to a Bose symmetric parametrization. In other words, the chiral Schwinger model may have any $a \geq 1$, but if two such models with opposite chiralities are soldered to yield the vector Schwinger model, then the minimal bound is the unique choice. Interestingly, the case $a = 1$ implies a massless mode in the chiral Schwinger model. The soldering mechanism therefore generates the massive mode of the Schwinger model from a fusion of the massless modes in the chiral Schwinger models.

We have therefore explicitly derived expressions for the chiral determinants (13) which simultaneously preserve the factorization property (6) and gauge invariance of the vector determinant. It was also perceived that the naive way of interpreting the chiral determinants as $W[A_+, 0]$ or $W[0, A_-]$ led to the supposed incompatiblity of factorization with gauge invariance since it missed the crossing graphs. Classically these graphs do vanish ($P_+P_- = 0$) so that it becomes evident that this incompatibility originates from a lack of properly accounting for the quantum effects. It is possible to interpret this effect, as we will now show, as a typical quantum mechanical interference phenomenon, closely paralleling the analysis in Young's double slit experiment. As a bonus, we provide a new interpretation for the Polyakov-Wiegman [2] identity. Rewriting (5) in Fourier space as

$$W[A_+, A_-] = -\frac{N}{2} \int d^2k \, \{A_+^*(k) \frac{k_-}{k_+} A_+(k) + A_-^*(k) \frac{k_+}{k_-} A_-(k) - 2A_+^*(k) A_-(k)\}$$

$$= \quad -\frac{N}{2}\int d^2k \mid \sqrt{\frac{k_-}{k_+}}A_+(k) - \sqrt{\frac{k_+}{k_-}}A_-(k) \mid^2 \tag{25}$$

immediately displays the typical quantum mechanical interference phenomenon, in close analogy to the optical example,

$$W[A_+, A_-] = -\frac{N}{2}\int d^2k \, \Big(\mid \psi_+(k) \mid^2 + \mid \psi_-(k) \mid^2 + 2\cos\theta \psi_+^*(k)\psi_-(k) \Big) \tag{26}$$

with $\psi_\pm(k) = \sqrt{\frac{k_\mp}{k_\pm}}A_\pm(k)$ and $\theta = \pm\pi$, simulating the roles of the amplitude and the phase, respectively. Note that in one space dimension, these are the only possible values for the phase angle θ between the left and the right movers. The dynamically generated mass arises from the interference between these movers, thereby preserving gauge invariance. Setting either A_+ or A_- to vanish, destroys the quantum effect, very much like closing one slit in the optical experiment destroys the interfernce pattern. Although this analysis was done for the abelian theory, it is straitghtforward to perceive that the effective action for a nonabelian theory can also be expressed in the form of an absolute square (25), except that there will be a repetition of copies depending on the group index. This happens because only the two-legs graph has an ultraviolet divergence, leading to the interference (mass) term. The higher legs graphs are all finite, and satisfy the naive factorization property.

It is now simple to see that (25) represents an abelianized version of the Polyakov Wiegman identity by making a familiar change of variables,

$$\begin{aligned} A_+ &= \frac{i}{e}U^{-1}\partial_+U \\ A_- &= \frac{i}{e}V\partial_-V^{-1} \end{aligned} \tag{27}$$

where, in the abelian case, the matrices U and V are given as,

$$U = \exp\{i\varphi\} \quad ; \quad V = \exp\{-i\rho\} \quad ; \quad UV = \exp\{i\Phi\} \tag{28}$$

with Φ being the gauge invariant soldered field introduced in (23). It is possible to recast (25), in the coordinate space, as

$$W[UV] = W[U] + W[V] + \frac{1}{2\pi}\int d^2x \left(U^{-1}\partial_+ U\right)\left(V\partial_- V^{-1}\right) \tag{29}$$

which is the Polyakov-Wiegman identity, satisfying gauge invariance. The result can be extended to the nonabelian case since, as already mentioned, the nontrivial interference term originates from the two-legs graph which has been taken into account. It is now relevant to point out that the important crossing piece in either (25) or (29) is conventionally [2, 1] interpreted as a contact (mass) term, or a counterterm, necessary to restore gauge invariance. In our analysis, on the contrary, this term was uniquely specified from the interference between the left and right movers in one space dimension, automatically providing gauge invariance. This is an important point of distinction.

To conclude, our analysis clearly revealed that no obstruction to gauge invariance is posed by the chiral decomposition of the 2D fermionic determinant. The claimed obstruction actually results from an incorrect interpretation of the chiral determinants. Bose symmetry gave a precise way of making sense of these determinants which were explicitly computed by considering the dual descriptions of decomposition as well as soldering. The close interplay between Bose symmetry and gauge invariance was illustrated in both these ways of looking at the fermionic determinant. At the dynamical level it was also shown how this symmetry is instrumental in fusing the massless modes of the left and right chiral Schwinger models to yield the single massive mode of the vector Schwinger model. Indeed it was explicitly shown that this mass generation is the quantum interference effect between the two chiralities, closely

resembling the corresponding effect in the double slit optical experiment. This led us to provide a novel interpretation of the Polyakov-Wiegman identity.

Our analysis indicated that the $a = 1$ regularisation for the determinant of the Chiral Schwinger Model was important leading to interesting effects. This parametrisation was also found to be useful in a different context [15]. On the other hand much of the usual analyses is confined to the $a = 2$ sector [16].

Finally, to put this work in a proper perspective it may be useful to once again remind the importance of Bose symmetry. It is an essential ingredient in getting the classic ABJ anomaly from the triangle graph [9, 10]. Just imposing gauge invariance on the vector vertices does not yield the cherished result. Bose symmetry coupled with gauge invariance does the job. This symmetry also played a crucial role in providing a unique structure for the 1-cocycle that is mandatory for smooth bosonization[5, 3]. It is therefore not surprising that Bose symmetry provided the definite guideline in preserving the compatibility between gauge invariance and chiral decomposition or soldering.

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APPENDIX

Here we explicitly show the soldering mechanism in the nonabelian context. The expressions for the chiral determinants analogous to (14) are given by [17],

$$\begin{aligned} W_+[g] &= I^{(-)}_{wzw}[g] - \frac{ie}{2\pi}\int d^2x tr(A_+ g^{-1}\partial_- g) - \frac{e^2 a}{4\pi}\int d^2x tr(A_+ A_-) \\ W_-[h] &= I^{(+)}_{wzw}[h] - \frac{ie}{2\pi}\int d^2x tr(A_- h^{-1}\partial_+ h) - \frac{e^2 b}{4\pi}\int d^2x tr(A_+ A_-) \end{aligned} \tag{30}$$

where the Wess-Zumino-Witten functional at the critical point ($n = \pm 1$) is given by (for details and the original papers, see [1]),

$$I^{\pm}_{wzw}[k] = \frac{1}{4\pi}\int d^2x tr(\partial_+ k \partial_- k^{-1}) \mp \frac{1}{12\pi}\Gamma_{wz}[k] \quad ; k = g, h \tag{31}$$

with the familiar Wess-Zumino term defined over a 3D manifold with the two-dimensional Minkowski space-time as its boundary,

$$\Gamma_{wz}[k] = \int d^3x \epsilon^{lmn}\, tr(k^{-1}\partial_l k\, k^{-1}\partial_m k\, k^{-1}\partial_n k) \tag{32}$$

In the above equations g and h are the elements of some compact Lie group and the parameters a and b manifest the regularisation or bosonisation ambiguities. Let us next consider the gauging of the global right and left chiral symmetries analogous to (15),

$$\begin{aligned} \delta g &= \omega g \\ \delta h &= h\omega \\ \delta A_\pm &= 0 \end{aligned} \tag{33}$$

where ω is an infinitesimal element of the algebra of the corresponding group. Note the order of ω which occurs once from the left and once from the right to properly

account for the two chiralities. In the abelian example, this just commutes and the ordering is unimportant leading to a unique transformation in (15). Under (33), the relevant variations are found to be,

$$\begin{aligned}\delta W_+[g] &= \int d^2x tr(\partial_-\omega J_+(g)) \\ \delta W_-[h] &= \int d^2x tr(\partial_+\omega J_-(h))\end{aligned} \tag{34}$$

where,

$$J_\pm = \frac{-1}{2\pi}\Big(\partial_\pm k k^{-1} + iekA_\pm k^{-1}\Big) \tag{35}$$

Now introduce the soldering field $B_\pm$ which transforms as,

$$\delta B_\pm = \partial_\pm\omega - [B_\pm, \omega] \tag{36}$$

whose abelian version just corresponds to (20). Then it may be checked that the following effective action,

$$W[g,h] = W_+[g] + W_-[h] - \int d^2x tr\Big(B_- J_+(g) + B_+ J_-(h) + \frac{1}{2\pi}B_+B_-\Big) \tag{37}$$

is invariant under the complete set of transformations. The auxiliary soldering field is eliminated, as usual, in favour of the other variables, by using the equations of motion,

$$B_\pm = -2\pi J_\pm(k) \tag{38}$$

The soldered effective action directly follows from (37) on substituting this solution,

$$\begin{aligned}W[G] &= I^+_{wzw}[G] + \frac{ie}{2\pi}\int d^2x tr\Big(A_- G^{-1}\partial_+ G - A_+\partial_- G G^{-1}\Big) \\ &- \frac{e^2}{2\pi}\int d^2x tr\Big(A_+ G A_- G^{-1} - \frac{a+b}{2}A_+A_-\Big)\end{aligned} \tag{39}$$

where $G = g^{-1}h$. Once again gauge invariance under the conventional set of transformations in which $A_\pm$ changes as a potential, is recovered only if $a+b=2$, exactly

as happened in the abelian case. Imposing Bose symmetry leads to the unique choice $a = b = 1$, completely determining the structure for the separate chiral components. Incidentally, by including the Yang-Mills term to impart dynamics so that these models become chiral QCD_2, it was found that unitarity could be preserved only for $a, b \geq 1$ [17]. Coupled with the above noted restriction, this leads to the Bose symmetric parametrisation. It is easy to see that (39) reduces to the abelian result (22) by setting $G = \exp(i\Phi)$. Observe that the soldering was done among the chiral components having opposite critical points. Any other combination would fail to reproduce the gauge invariant result. Indeed the gauge invariant effective action, being a functional of $G = g^{-1}h$, can be obtained by soldering effective actions (which are functionals of g and h) with opposite criticalities since changing $g \to g^{-1}$ converts the Wess-Zumino-Witten functional from one criticality to the other. The relevance of opposite criticality was also noted in another context involving smooth nonabelian bosonisation [4].

This nonabelian exercise, however, clearly reveals that the physics of the problem of chiral soldering (or decomposition) and the role of Bose symmetry is contained in the abelian sector. The rest is a matter of technical detail. Indeed, following similar steps, it is also possible to discuss chiral decomposition for the nonabelian case and obtain identical conclusions.

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